Let $n > 5$.
Let $\{ p_1, p_2, ..., p_k\}$ be the set of all primes $\leq n$, $2 = p_1 < p_2 < ... < p_k$.

Assume $n! = p_1^{a_1} p_2^{a_2} \ ... \ p_k^{a_k}$ is the prime factorization of $n!$.
By the definition of the Lambda function λ, we have

$$\lambda(n!) = \text{LCM}(\lambda(p_1^{a_1}), \lambda(p_2^{a_2}), \ ..., \lambda(p_k^{a_k})) = \text{LCM}(2^{a_1-2}, (p_2-1)p_2^{a_2-1}, ..., (p_k-1)p_k^{a_k-1})$$

By the definition of the LCM, $\lambda(n!)$ = the product of the primes q that appear in the numbers $(p_i-1)p_i^{a_i-1}$ with exponent the maximum of the exponents to which q appears in each of these numbers.
Clearly, q cannot be anything else but one of the primes $2, p_2, p_3, ..., p_k$.
First, let $q \neq 2$, say $q = p_i$. It appears in the number $(p_i-1)p_i^{a_i-1}$ with exponent $a_i - 1$.
Can it appear in one of the other numbers $(p_j-1)p_j^{a_j-1}$ with a higher exponent ?
If yes, it would be a factor of $p_j - 1$. But, $n!$ is divisible by the product of all the numbers $p_j - 1$. The contribution of that product to the factor $p_i^{a_i}$ is of course strictly less than $p_i^{a_i}$.
Thus, p_i cannot be in $p_j - 1$ with an exponent greater than $a_i - 1$.
If $q = 2$, the same argument is valid, since we assumed $n > 5$. In that case, there are more even numbers than there are numbers of the form $p_i - 1$, i.e : the highest power to which 2 enters in $p_i - 1$ can never exceed $a_1 - 2$.
Thus, $\lambda(n!) = 2^{a_1-2} p_2^{a_2-1} \ ... \ p_k^{a_k-1}$, precisely Sebastian's statement.

Let's give an example : $n = 10$.
Then : $10! = 2^8 \cdot 3^4 \cdot 5^2 \cdot 7$
Hence : $\lambda(n!) = \text{LCM}(\lambda(2^8), \lambda(3^4), \lambda(5^2), \lambda(7)) = \text{LCM}(2^6, (3-1)3^3, (5-1)5, 7-1)$.
$q = 3$: the highest power of 3 in $\lambda(10!)$ is 3 since 3 cannot enter in $5-1$ or $7-1$ with a higher power than 3.
$q = 5$: the highest power of 5 in $\lambda(10!)$ is 1 since 5 cannot enter in $3-1$ or $7-1$ with a higher power than 1.
$q = 7$: the highest power of 7 in $\lambda(10!)$ is 0 since 7 cannot enter in $5-1$ or $7-1$ with a higher power than 0.
$q = 2$: the highest power of 2 in $\lambda(10!)$ is 6 since 2 cannot enter in $3-1, 5-1$ or $7-1$ with a higher power than 6.
(the last reasoning would not be valid if n was equal to 5, therefore I excluded it in advance).
Thus, the value of $\lambda(n!)$ is $2^6 \cdot 3^3 \cdot 5$.

Let's summarize this by saying that Sebastians's formula is valid because the a_i are big enough to ensure q^a not to divide $p-1$ whenever p, q are primes $\leq n$.