Addendum to Conjecture 50, by Patrick Capelle.
November, 17, 2006.

I would like to present here a little work which can be useful to highlight the importance of these new conjectures. The text below is a logic prolongation of my recent contribution of 13 october 2006 (see ‘Conjecture 49’).

I must confess first that conjecture B is my favorite.
I am fascinated by its simplicity.
It is so simple that i am astonished that it was not discovered before.
Conjecture B is with the previous conjecture B (see 'Conjecture 49') what the Generalized de Polignac conjecture is with the 'strong' Goldbach conjecture.
The primes are replaced here by the semiprimes, and for the results the even numbers are replaced by the integers.
There is another important reason to explain my interest for the conjecture B :
Every integer, without exception, appears as the determinant of an 2 x 2 matrix of prime numbers in infinitely many ways.
I cannot realize yet all the algebraic implications of this exceptional situation. Determinants have a geometrical meaning and many interesting properties. They are important both in calculus and in multilinear algebra. They are perhaps useful in the study of several conjectures mentioned here.
Note some interesting references (coming from the On-line Encyclopedia of Sequence Integers) :
A117027, A117301, A117329, A117330, A118799, A118815, A067549.
I wonder whether each integer can be written as the determinant of an n x n matrix of prime numbers when n is fixed and greater than 2 ...

Here also it is useful to introduce some (new) definitions :
1. An Extended Semiprime is a semiprime whose each factor is equal to a prime number or 1.
It means that an Extended Semiprime is a semiprime, a prime number or 1.
2. An Extended Prime is a number which is prime or 1.
Note that an Extended Semiprime is either an Extended Prime or the product of two Extended Primes.
3. Twin semiprimes are pairs of semiprimes of the form (s, s + 1).
In other words, they are pairs of semiprimes which differ by 1.

Examples using the definition of Extended Semiprimes and Extended Primes :
1. Every integer is the difference of two Extended Semiprimes in infinitely many ways. It is another formulation of conjecture A.
2. Every Extended Semiprime is the difference of two Extended Semiprimes in infinitely many ways. It is a consequence of conjecture A.
3. Every natural number, different from 0, either is itself an Extended Prime, or can be written as a product of Extended Primes. Every natural number, different from 0, either is itself an Extended Semiprime, or can be written as a product of Extended Semiprimes. It would take too much time to discuss here about these two ideas. Note that the number 1 is often excluded from the list of primes, and one reason for this exclusion is the uniqueness part of the fundamental theorem of arithmetic. This last theorem states that every integer (except the number 1) either is itself a prime number, or can be written apart from rearrangement as a unique product of prime numbers.

The conjectures B and C are particular cases of conjecture A.
Other conjectures/theorems are also connected to particular cases of conjecture A :
1. $p = 1, q > 1, r = 1, s > 1$: every even number 2n is the difference of two consecutive primes in infinitely many ways (de Polignac's conjecture).
This conjecture is equivalent to the statement "every even number 4n is the difference of two consecutive even semiprimes in infinitely many ways" [1].
Moreover, de Polignac's conjecture implies that every even number > 2 is the difference of two semiprimes in infinitely many ways [2].
If one extends the reasoning to the negative even numbers, we can see that nearly all the even integers of conjecture B are concerned !

2. $p = 1, q > 1, r = 1, s > 1$: every even number is the difference of two primes in infinitely many ways (generalization of de Polignac's conjecture).
This conjecture is equivalent to the statement "every even number 4n is the difference of two even semiprimes in infinitely many ways" [3].
Moreover, the Generalized de Polignac conjecture implies that every even number > 2 is the difference of two semiprimes in infinitely many ways [4].
In conjecture B (again), after an extension of the reasoning to the negative even numbers, we can see that nearly all the even integers are concerned !

3. $p = 1, q > 2, r = 1, s > 2, n = 2$: there are infinitely many primes s such that $q = s + 2$ is also prime (Twin Prime conjecture).
This conjecture is a particular case of de Polignac's conjecture, but also a particular case of Generalized de Polignac conjecture.
It is equivalent to the statement "there are infinitely many pairs of even semiprimes which differ by 4" [5].
Moreover, the twin prime conjecture implies that every even semiprime is the difference of two semiprimes in infinitely many ways [6].
All the even semiprimes of conjecture B are concerned !
Note that Viggo Brun showed that the sum of reciprocals of the twin primes converges to a finite value, now called Brun's constant [7].
Note also that the first Hardy-Littlewood conjecture, called the k-tuple conjecture, is a generalization of the twin prime conjecture ...

4. $q >1, r = 1, s > 1, n = 2$: there are infinitely many primes s such that s + 2 is either a prime or a semiprime (Chen's theorem). The primes s are called Chen primes. A closely related theorem of Chen Jingrun asserts that every sufficiently large even number is the sum of a prime and another number which is either a prime or a semiprime. These results are in connection with the twin prime conjecture and the strong Goldbach's conjecture respectively.

5. $p = 1, q > 1, r = 1, s > 1$: there is a constant $c < 1$ and infinitely many primes s such that $q - s < c.\ln s$, where q is the next prime after s (Erdös, 1940). This result was successively improved : $c < 0,25$ (Helmut Maier, 1986), then c could be improved further to 0,085786...(Daniel Goldston and Cem Yildirim, 2004). Finally Daniel Goldston, Janos Pintz and Cem Yildirim showed in 2005 that c could be chosen arbitrarily small. Assuming the Elliott-Halberstam conjecture, they also proved that there are infinitely many pairs of primes which differ by 16 or less ($p = 1, q > 1, r = 1, s > 1, n <= 16$).

6. $p = 2, r = 3$: all the prime numbers are of the form $|2q - 3s|$, where q and s are primes or 1. Note that it is only in one way for the prime number 2 and in two ways for the prime number 3 (new Capelle's conjecture, found the 26 oct 2006, inspired by Papadimitriou's conjecture).

7. p = 2, q > 1, r = 3, s > 1 : all the prime numbers are of the form |2q - 3s|, where q and s are primes. Note that it is only in one way for the prime numbers 2 and 3 (new Capelle's conjecture, found the 26 oct 2006, inspired by Firoozbakht's conjecture) [8]. We can also say that all the odd prime numbers are of the form |2q - 3s|, where q and s are odd primes (p = 2, q > 2, r = 3, s > 2).

8. p = 2, q > 2, r = 1 , s > 2 : all the odd integers are of the form 2q - s, where q and s are odd primes, in infinitely many ways (new Capelle's conjecture, found the 27 oct 2006, inspired by Levy's conjecture) [9].

9. p > 1, q > 1, r = 1 , s > 1 : every integer n, different from 0, can be written in infinitely many ways as n = pq - s, where p, q, s are distinct primes (new Capelle's conjecture, found the 15 nov 2006, inspired by Zumkeller's conjecture [10]).

10. p = 1, q > 2, r = 1, s > 2, n = 2.t with t prime : every even semiprime is the difference of two odd primes in infinitely many ways (new Capelle's conjecture, found the 28 oct 2006). It is a particular case of the Generalized de Polignac conjecture [11].

11. p = 1, q > 2, r = 1, s > 2, q <> s : every even number is the difference of two t-primes, where a t-prime is a prime which has a twin, in infinitely many ways (new Capelle's conjecture, found the 29 oct 2006, inspired by Dubner's conjecture [12]). Starting with this idea, some generalizations can be proposed [13].

12. p > 1, q > 1, r >1, s > 1 : there are infinitely many semiprimes rs such that pq = rs + 1 is also semiprime (new Capelle's conjecture, which could be called the "Twin semiprime conjecture"). Twin semiprimes are pairs of semiprimes which differ by 1. This conjecture is in fact a particular case of the conjectures B and C.

13. p = 2, q > 1, r = 2, s > 1 : there are infinitely many pairs of even semiprimes which are consecutive semiprimes (new Capelle's conjecture, found the 15 nov 2006) [14]. I found 190 pairs [15] and 13 triplets [16] among the first 10000 semiprimes.

As you can see, i took some statements mentioned in my contribution of friday 13 oct 2006 (see http://www.primepuzzles.net/conjectures/conj_049.htm), transformed them into conjectures by a distorting mirror and often added the expression "in infinitely many ways".
The general context of the conjecture A gives the opportunity to propose new ideas ...

[1] For each n, the even number 2n is the difference of two consecutive primes in infinitely many ways.
==> For each n, there exists an infinitity of pairs of consecutive primes s_i and q_i ,
with i natural number and q_i > s_i , such that 2n = q_1 - s_1 = q_2 - s_2 = ...
==> 2.2n = 2.(q_1 - s_1) = 2.(q_2 - s_2) = ...
==> 4n = 2.q_1 - 2.s_2 = 2.q_2 - 2.s_2 = ...
==> 4n is the difference of two consecutive even semiprimes in infinitely many ways.

[2] For each n, the even number 2n is the difference of two consecutive primes in infinitely many ways.
==> For each n, there exists an infinitity of pairs of consecutive primes s_i and q_i ,
with i natural number and q_i > s_i , such that 2n = q_1 - s_1 = q_2 - s_2 = ...
==> t.2n = t.(q_1 - s_1) = t.(q_2 - s_2) = ..., with t prime.
==> 2m = t.q_1 - t.s_2 = t.q_2 - t.s_2 = ..., with m = t.n

m covers all the natural numbers > 1 when t prime and n natural number (think to the sieve of Eratosthenes).
==> Every even number 2m, with m > 1, is the difference of two semiprimes in infinitely many ways.
In conjecture B, all the even natural numbers > 2 are concerned !

[3] For each n, the even number 2n is the difference of two primes in infinitely many ways.
==> For each n, there exists an infinitity of pairs of primes s_i and q_i , with i natural number and q_i > s_i , such that 2n = q_1 - s_1 = q_2 - s_2 = ...
==> 2.2n = 2.(q_1 - s_1) = 2.(q_2 - s_2) = ...
==> 4n = 2.q_1 - 2.s_2 = 2.q_2 - 2.s_2 = ...
==> 4n is the difference of two even semiprimes in infinitely many ways.

[4] For each n, the even number 2n is the difference of two primes in infinitely many ways.
==> For each n, there exists an infinitity of pairs of primes s_i and q_i , with i natural number and q_i > s_i , such that 2n = q_1 - s_1 = q_2 - s_2 = ...
==> t.2n = t.(q_1 - s_1) = t.(q_2 - s_2) = ..., with t prime.
==> 2m = t.q_1 - t.s_2 = t.q_2 - t.s_2 = ..., with m = t.n
m covers all the natural numbers > 1 when t prime and n natural number (think to the sieve of Eratosthenes).
==> Every even number 2m, with m > 1, is the difference of two semiprimes in infinitely many ways.
In conjecture B, all the even natural numbers > 2 are concerned !

[5] There are infinitely many primes s such that q = s + 2 is also prime (Twin Prime conjecture).
==> There exists an infinitity of pairs of primes s_i and q_i , with i natural number and q_i > s_i , such that 2 = q_1 - s_1 = q_2 - s_2 = ...
==> 2.2 = 2.(q_1 - s_1) = 2.(q_2 - s_2) = ...
==> 4 = 2.q_1 - 2.s_2 = 2.q_2 - 2.s_2 = ...
==> There are infinitely many pairs of even semiprimes which differ by 4.

[6] There are infinitely many primes s such that q = s + 2 is also prime (Twin Prime conjecture).
==> There exists an infinitity of pairs of primes s_i and q_i , with i natural number and q_i > s_i such that 2 = q_1 - s_1 = q_2 - s_2 = ...
==> t.2 = t.(q_1 - s_1) = t.(q_2 - s_2) = ..., with t prime.
==> 2.t = t.q_1 - t.s_2 = t.q_2 - t.s_2 = ..., with t prime.
==> Every even semiprime is the difference of two semiprimes in infinitely many ways.

[7] Is the sum of reciprocals of 'twin semiprimes' (i.e., consecutive semiprimes such that the difference is 1) also convergent ? The number of 'twin semiprimes' less than N does not exceed C.f(N) for some absolute constant C > 0 ?

[8] This new conjecture leads to some interesting generalizations :
1. All the odd integers are the difference of an even semiprime and an odd semiprime in infinitely many ways (p = 2, q > 1, r > 2, s > 2).
In conjecture B, all the odd integers are concerned !
2. Every k-almost prime, with k > 0, can be written as an absolute difference of two (k+1)-almost primes.

[9] It means that all the odd prime numbers are of the form |2q - s| in the same time that they are of the form |2q -3s|, with q and s odd primes in both cases.

[10] See http://www.research.att.com/~njas/sequences/A100952

[11] Note that by conjecture B, all the semiprimes are the difference of two semiprimes in infinitely many ways. It implies that every even semiprime is at the same time, in infinitely many ways, the difference of two odd primes and the difference of two semiprimes. In other words, there are at least two infinite ways of expression of the even semiprimes as difference of two 'Extended Semiprimes'.

[12] Dubner's conjecture states that every even number greater than 4208 is the sum of two t-primes, where a t-prime is a prime which has a twin. This conjecture implies the Twin prime conjecture (because it would imply an infinite number of t-primes, and thus an infinite number of twin prime pairs) and the 'strong' Goldbach's conjecture (because it has already been verified that all the even numbers 2n, such that 2 < 2n <= 4208, are the sum of two primes).
Dubner's conjecture can be generalized as follows :
1. For each natural number k > 0, every sufficiently large even number n(k) is the sum of two d(2k)-primes, where a d(2k)-prime is a prime p which has a prime q such that d(p,q) = |q - p| = 2k and p, q successive primes. The conjecture will imply the 'strong' Goldbach's conjecture (for all the even numbers greater than a large value l(k)) for each k, and the de Polignac's conjecture if we consider all the cases k. Note that the case k = 1 corresponds to Dubner's conjecture.
2. The same idea, but p and q are not necessarily consecutive in the definition of a d(2k)-prime.We find again the Dubner's conjecture as a particular case (k = 1). If we look at the implications, the 'strong' Goldbach's conjecture and the Generalized de Polignac's conjecture (if we consider all the cases k) are concerned.

[13]
1. For each natural number k > 0, every even number n(k) is in infinitely many ways the difference of two d(2k)-primes, where a d(2k)-prime is a prime p which has a prime q such that d(p,q) = |q - p| = 2k and p, q successive primes.
2. The same idea, but p and q are not necessarily consecutive in the definition of a d(2k)-prime.

[14] If q_i and s_i are consecutive primes, it is false to say that 2.q_i and 2.s_i are consecutive semiprimes. They are consecutive even semiprimes, but in general not consecutive semiprimes. However, i guess that there are infinitely many pairs of consecutive even semiprimes which are consecutive semiprimes.
On the other hand, if 2.q_i and 2.s_i are consecutive semiprimes, then q_i and s_i are consecutive primes. It is also the case when 2.q_i and 2.s_i are even semiprimes which differ by 4 (note that they are not necessarily consecutive semiprimes) : q_i and s_i will be consecutive primes (more precisely they will be twin primes).
Note that (4,6) is the only pair of even semiprimes which differ by 2.

[15] Pairs of even semiprimes (among the first 10000 semiprimes) which are consecutive semiprimes : (4,6), (10,14), (58,62), (454,458), (458,466), (538,542), (614,622), (1082,1094), (1234,1238), (1318,1322), (1478,1486), (1618,1622), (1718,1726), (1874,1882), (2062,2066), (2374,2386), (2554,2558), (2846,2854), (2902,2906), (3574,3578), (3722,3734), (3998,4006), (4174,4178), (4258,4262), (4474,4478), (4946,4954), (5098,5102), (5414,5422), (5422,5426), (5498,5506), (6334,6338), (6598,6602), (6658,6662), (6686,6694), (6718,6722), (6778,6782), (6914,6922), (6922,6926), (7054,7058), (7346,7354), (7534,7538), (7642,7646), (7702,7706), (7754,7762), (7838,7846), (8038,8042), (8098,8102), (8422,8434), (8434,8438), (8674,8678), (8818,8842), (8962,8966), (9038,9046), (9274,9278), (9278,9286), (9314,9326), (9458,9466), (9578,9586), (10042,10046), (10334,10342), (10454,10462), (10462,10466), (10826,10834), (10834,10838), (10882,10886), (10954,10958), (11002,11006), (11054,11062), (11114,11126), (11314,11318), (11486,11498), (11698,11702), (12094,12106), (12262,12266), (12394,12398), (12538,12542), (12542,12554), (12718,12722), (12898,12902), (12938,12946), (13522,13526),

(13582,13586), (13714,13726), (13994,14002), (14078,14086), (14422,14426), (14662,14666), (15278,15286), (15374,15382), (15578,15586), (15734,15746), (16022,16034), (16222,16234), (16334,16342), (16582,16586), (16886,16894), (17074,17078), (17254,17258), (17378,17386), (17386,17398), (17482,17494), (17726,17734), (17938,17942), (18682,18686), (19258,19262), (19478,19486), (20182,20186), (20354,20362), (20542,20546), (20674,20686), (20854,20858), (20918,20926), (20998,21002), (21058,21062), (21194,21202), (21314,21326), (21562,21578), (22342,22346), (22478,22486), (22934,22942), (23354,23362), (23554,23558), (23666,23678), (23878,23882), (24014,24022), (24074,24082), (24086,24098), (24406,24422), (24478,24482), (24974,24982), (25514,25526), (25642,25646), (25778,25786), (26002,26006), (26014,26018), (26198,26206), (26242,26254), (26294,26302), (26434,26438), (26674,26678), (27746,27754), (27758,27766), (27802,27806), (27806,27814), (27994,27998), (28162,28166), (28346,28354), (28642,28646), (28814,28822), (29122,29126), (29182,29186), (29558,29566), (30274,30278), (30574,30578), (30658,30662), (31294,31298), (31322,31334), (31334,31342), (31474,31478), (32002,32014), (32114,32122), (32122,32126), (32138,32146), (32374,32378), (32378,32386), (32962,32974), (33298,33302), (33314,33322), (33382,33386), (34054,34058), (34934,34942), (34978,34982), (35314,35318), (35362,35366), (35494,35498), (35674,35678), (35914,35918), (35974,35978), (36082,36086), (36238,36242), (36914,36922), (37174,37186), (37586,37594), (38422,38426), (38842,38846), (38854,38858), (39154,39166), (39922,39926), (39982,39986), (40814,40822).

[16] Triplets of even semiprimes (among the first 10000 semiprimes) which are consecutive semiprimes : (454,458,466), (5414,5422,5426), (6914,6922,6926), (8422,8434,8438), (9274,9278,9286), (10454,10462,10466), (10826,10834,10838), (12538,12542,12554), (17378,17386,17398), (27802,27806,27814), (31322,31334,31342), (32114,32122,32126), (32374,32378,32386).