

Here's the proof of this test.

First let's prove that the product of the binomials being not divisible by N plus 1 and N itself are coprime. If N is prime it's obvious, since there aren't binomials not divisible. If instead N is composite so $N = pq$, with p prime, let's consider $\binom{N}{p}$. There're two important properties: it's divisible by $N/p = q$, and it's one of the test's binomials not divisible by N . In fact

$$\binom{N}{p} = \frac{N}{p} \binom{N-1}{p-1} = q \frac{(n-1)(n-2)\cdots(n-p+1)}{(p-1)!}$$

In order to be a multiple of N , the fraction must be a multiple of p , but the numerator is coprime since p doesn't divide any factor in it. Doing this for every prime number in the factorization of N , at the end of the day we get that the product of those binomials must be divisible by N . Therefore the product plus 1 and N are coprime.

Thus the test's fraction maintain N as denominator. Excluding finite decimal number and pre-period means that the denominator N must not be a multiple of 2 or 5, so it must be relatively prime with 10. The decimal period of a fraction with denominator N is equal to $\text{ord}_N(10)$, that is the minimum k such that

$$10^k \equiv 1 \pmod{N}$$

Consequentially, if the period k is a divisor of $n-1$, raising to power we get

$$10^{N-1} \equiv 1 \pmod{N}$$

i.e. the definition of Fermat-pseudoprime to base 10.